Exhibit G (i)

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A Test of DLOM Computational Models

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Abstract

The purpose of this article is to provide information concerning the viability of five mathematical models for use in determining the discount for lack of marketability. Based on a comparison to data from the FMV Restricted Stock Study, results from the Chaffe, Finnerty, and Meulbroek models result in discounts within the range of the data for one- and two-year periods, but results from the Longstaff and Tabak models do not. In addition, the Meulbroek model produces results that behave over time in a reasonable way, but there are difficulties in the performance over time with each of the other models. However, the Meulbroek and Tabak models present a theoretical difficulty. Each of these models uses a return from the capital asset pricing model as a basis. As a result, each of these models would show no discount for a restricted security intended to track the market portfolio even though a discount may be theoretically appropriate in such a situation. Finally, all of these models used total volatility of return as an input without segregating the impact of growth on the volatility.

Introduction

In recent years a number of mathematical models have been developed to determine the reduction in value resulting from the inability to sell a publicly traded stock for a period of time. The purpose of this article is to provide information useful in judging the effectiveness of one class of these models. In this article the effectiveness of each of several mathematical models will be tested in two ways. First, the results of the models will be compared to real-world data. Second, the results will also be tested over time to determine whether the models appear to have theoretical problems and whether they appear to make common sense.

Models to Be Tested

I separate discount for lack of marketability (DLOM) computational models into three different classes: income approach models, regression models, and time/volatility models. Income approach models are those most compatible when an income approach has first been performed so that assumptions about the future are available. Two models that I include in this class are the Quantitative Marketability Discount Model and my time model.[1] Regression models are mathematical models that have been developed by a statistical analysis of restricted stock. Examples of such models include those developed by Trout, Silber, and Hertzel and Smith.[2] The time/volatility models are models developed following a theoretical construct resulting in a closed-form mathematical expression.

Although it would be desirable to test the income approach models by comparison to a database of transactions, these models rest on judgments about the future of the security, and they are difficult to compare to transactional data in which information about future judgments applicable to the transactions is not available. It would also be desirable to test the regression models. However, these models are each based on a number of variables, some of which are different from model to model. Some of these variables are difficult to easily identify from existing databases. These factors make it difficult to test these regression models at this time. Accordingly, these two classes of models are not tested in this article.

The time/volatility models are of increasing interest to valuators, and some indication of their effectiveness would be useful to practitioners. Each of the models in this class has two common variables, time and volatility. Further, they have few or no other variables. Accordingly, the time/volatility class of models can be meaningfully studied assuming availability of a database including these variables. This class of mathematical models is the focus of this article.

The models to be tested will be those presented by Chaffe, Finnerty, Longstaff, Meulbroek, and Tabak.[3] Each of these models presents a theory that can be reduced to a closed-form expression to compute a dis-

[1] For the Discount Model, see Z. Christopher Mercer, *Quantifying Marketability Discounts* (Memphis: Peabody Publishing, 1997). For the time model, see John J. Stockdale, "Time Is of the Essence: A Proposed Model for Computing the Discount for Lack of Marketability," *Business Valuation Review* 25 (Fall 2006):108–113.

[2] Robert R. Trout, "Estimation of the Discount Associated with the Transfer of Restricted Securities," *Taxes—The Tax Magazine* (June 1977):381–385. William L. Silber, "Discounts on Restricted Stock: The Impact of Illiquidity on Stock Prices," *Financial Analysts Journal* (July–August 1991):60–61. Michael Hertzel and Richard Smith, "Market Discounts and Shareholder Gains for Placing Equity Privately," *Journal of Finance* 48 (June 1993):459–485.

[3] David B. H. Chaffe III, "Option Pricing as a Proxy for Discount for Lack of Marketability in Private Company Valuations—A Working Paper," *Business Valuation Review* (December 1993):182–188. John D. Finnerty, "The Impact of Transfer Restrictions on Stock Prices," paper presented at the 22nd Annual Advanced Business Valuation Conference, Chicago, Ill., October 2003. Francis A. Longstaff, "How Much Can Marketability Affect Security Values?" *Journal of Finance* (December 1995):1767–1774. Lisa K. Meulbroek, "Company Stock in Pension Plans: How Costly Is It?" Harvard Business School Working Paper 02–058, Cambridge, Mass. David I. Tabak, " A CAPM-Based Approach to Calculating Illiquidity Discounts," unpublished manuscript, NERA Economic Consulting, New York, NY.

count from a publicly traded price. Each of the models uses time and volatility (standard deviation of return of the security) as inputs.[4] The Meulbroek and Tabak models also use capital asset pricing model (CAPM) beta and standard deviation of return of the market as inputs. Finnerty and Chaffee also use the risk-free rate and dividend rate as an input. It is my assertion that if the models have practical application for the valuator in computing the DLOM, they should produce a result that is at least somewhat representative of the real world even though they might not precisely predict a discount for every situation. Accordingly, I test the models by comparing their results to data from a restricted stock study using standard deviation of return and time as points of comparison for each transaction.

Robak has previously investigated the performance of the Finnerty, Longstaff, and Tabak models.[5] He used the proprietary LiquiStat database, which contains transactions on the resale of restricted stock in private transactions. Based on a data set containing forty-one trades, he found that none of these three models performed well compared to his data set, which involved high-volatility stocks. The Longstaff and Tabak models generated discounts that were consistently too high, and the Finnerty model generated discounts that were consistently too low.

Data Used for Comparison

To study the viability of the mathematical models, a source of data showing the amount of the discount at points that can be compared to the models is needed. The FMV Restricted Stock Study has been used.[6] This study is the only restricted stock study available to the practitioner that provides detailed data for a number of factors for each of the transactions in the study. It also provides a substantial number of transactions. As a result of these factors, this study is the best study available to practitioners for statistical study of private placements. Another option would have been pre–initial public offering (IPO) studies. However, these studies by their nature cannot provide a volatility statistic because the stock was not public at the time of the pre-IPO transaction, and there is no history of trading in the stock prior to the IPO.

The transactions from the FMV Restricted Stock Study are shown as the diamond-shaped data points in Figures 1 and 2. The data have been broken into two groups: transactions occurring before 1997 and after 1996. The pre-1997 graph includes 240 transactions. One transaction from the database was excluded for presentation purposes because it had a very high volatility. The post-1996 graph includes 223 transactions. Four transactions were excluded for presentation purposes because they had relatively large premiums (greater than 25%). Early in 1997 the SEC Rule 144 holding period was changed from two years to one year. Accordingly, pre-1997 results roughly represent a two-year restriction period, and the post-1996 data roughly represent a one-year restriction period.[7] As shown in the figures, there is considerable scatter in the data. A regression analysis shows a statistical correlation between the discount and the standard deviation of return (at the 1% level), but the R^2 (coefficient of determination) is relatively low (about 10%). Because of the significant scatter of the data, statistical analyses of results of the models are not performed. Instead, graphical comparisons are used to study the reasonableness of model results.

Description of the Models

Chaffe

This mathematical model is based on the observation that a put option represents the value of a right to sell a stock. Because a discount for lack of marketability results from an inability to exercise a right to sell, this model measures the discount by dividing the value of the put computed using a Black-Scholes model at the time period of restriction by the current value of the stock.

Finnerty

In the Finnerty model, the discount is modeled as a function of restricted transferability using the value of an average strike put option. Among other assumptions, this model assumes that the shareholder has no special

[4] There is one other model I place in the time/volatility class, which was developed by Kahl, Liu, and Longstaff. I have excluded this model because it is somewhat difficult to use. The paper developing it does not reduce it to a closed-form mathematical expression. Among the required inputs are an investor's risk coefficient and the investor's fraction of wealth held. These variables are difficult to use in the context of a hypothetical buyer and seller. See Mathias Kahl, Jun Liu, and Francis A. Longstaff, "Paper Millionaires: How Valuable Is Stock to a Restricted Shareholder Who Is Restricted from Selling It?" Workpaper 9–01, UCLA Anderson Graduate School of Management, Los Angeles.

[5] Espen Robak, "Lemons or Lemonade? A Fresh Look at Restricted Stock Discounts," *Valuation Strategies* 10 (January/February 2007):4–15 and 46–48.

[6] The FMV Restricted Stock Study is a proprietary study available for purchase from Business Valuation Resources at www.bvmarketdata.com. I have no relationship whatsoever, financial or otherwise, with the developers of the FMV database, and I have no relationship with the publisher of the FMV database other than as a customer.

[7] The effective restriction period could be longer as a result of dribble-out rules. The median block size of the pre-1997 data is about 11%, and the median block size of the post-1996 data is about 9%. As a result, the effective period of restriction could be substantially longer than the required minimum holding period. However, the effective restriction period could also be shorter because some of the transactions included registration rights.

market timing ability. Such special market timing ability is an assumption in the Longstaff model. The formula for this model is as follows:

$$D(T) = V_0\left[e^{(r-q)T}N\left(\frac{r-q}{v}\sqrt{T}+\frac{1}{2}v\sqrt{T}\right)\right.$$
$$\left.-N\left(\frac{r-q}{v}\sqrt{T}-\frac{1}{2}v\sqrt{T}\right)\right],$$
$$v^2 = \sigma^2 T + \ln\left[2\left(e^{\sigma^2 T} - \sigma^2 T - 1\right)\right] - 2\ln\left[e^{\sigma^2 T} - 1\right],$$

where

r = risk-free rate
q = dividend rate
$N(–)$ = cumulative normal distribution function
v = value of the security
T = time of restriction
σ^2 = the statistical variance of return of the security
e = exp = a mathematical constant = 2.71828 . . .

Longstaff

In the Longstaff model, the discount is a function of restricted transferability that can be measured using standard option-pricing theory. One assumption made in this model is that the holder has special market timing ability but cannot benefit from this ability during the restriction period. The model is stated to result in an upper bound for percentage discounts for lack of marketability.

The formula for this model is as follows:

$$F(V,T) = V\left(2+\frac{\sigma^2 T}{2}\right)N\left(\frac{\sqrt{\sigma^2 T}}{2}\right)$$
$$+ V\sqrt{\frac{\sigma^2 T}{2\pi}}\exp\left(-\frac{\sigma^2 T}{8}\right) - V,$$

where

$F(V,T)$ = the upper bound of the amount of the discount
$N(–)$ = cumulative normal distribution function
V = value of the security
T = time of restriction
σ^2 = the statistical variance of return of the security
exp = e = a mathematical constant = 2.71828 . . .

Meulbroek

This model was developed to investigate the cost of holding a single stock in a retirement plan as opposed to holding a more diversified portfolio. Accordingly, the underlying theory is that the DLOM is the result of being forced to carry extra risk in a single stock over and above that in a portfolio during the period in which the security cannot be sold. According to the author, this model produces a lower limit on the discount. She states, "This method . . . should yield an upper-bound on the private value of company stock to employees, because the actual value of company stock depends not only on the employees' level of diversification, but also on other specific risk preferences."[8]

The formula for this model may be stated as follows

$$\text{DLOM} = 1 - \left[1/(1+R)^N\right],$$

where

R = incremental rate of return from holding a single stock instead of a portfolio
N = the period of time the investor is forced to hold the stock before being able to diversify
R = market risk premium × (sigma s/sigma m − beta)
sigma s = standard deviation of return for the stock
sigma m = standard deviation of return of the market
beta = the CAPM beta of the stock
Market risk premium = the risk premium required for equity in excess of the risk-free rate.

It is worth noting that sigma s/sigma m is sometimes called the total beta.

The formula presented above is for a completely undiversified investor, that is, one who holds all wealth in the security being valued. There is a modified formula for an investor who is partially diversified. The formula for the partially diversified case has additional terms resulting from the need to determine the weighting of the stock held and the volatility of the portfolio. These additional factors are difficult to evaluate for a situation involving a hypothetical investor, be it a buyer or a seller. Accordingly, the formula tested in this article is for the completely undiversified case.

Because the model is based on a comparison of specific company returns to returns generated using the capital asset pricing model, this model has several limitations. First, it indicates no DLOM for cases where the difference between total beta and CAPM is zero. This situation could arise where an investor is holding a security in a company holding the market portfolio of stock even though it may be difficult for an investor to sell the security. For the market portfolio, the total beta = the standard CAPM beta, and thus the difference between them is zero.

Second, the computation of beta for an individual company using historical price and return data has certain difficulties. One problem is that beta may not be stable over time so that beta should not be determined at one particular time but should be studied over a period of time. In addition, beta computations based solely on

[8] Meulbroek, "Company Stock," 15.

fluctuations of return may not be statistically significant. That is, a regression analysis may indicate that there is a poor correlation between the company rate of return and the rate of return of the market. This situation commonly arises when computing beta. In such a situation, the Meulbroek model may understate the true discount. This model is based on the assumption that beta is a meaningful statistic. However, it can be shown that the discount computed using this model tends to increase as the correlation with the market decreases.

Tabak CAPM

The Tabak model works on the same general theory as the Meulbroek model. However, it uses the ratio of variance of company return to variance of market return instead of the standard deviation of return. Given the same data as the Meulbroek model, the Tabak model results in a higher DLOM.

The formula for the Tabak model is as follows:

$$\text{DLOM} = 1 - \exp^{-(\sigma_s^2/\sigma_m^2) \times RP \times T},$$

where

$\exp = e =$ a mathematical constant $= 2.71828\ldots$
$\sigma_s{}^2 =$ the statistical variance of return of the stock
$\sigma_m{}^2 =$ the statistical variance of return of the market
RP = the equity risk premium
$T =$ the time of restriction.

Description of the Results

The first test of the models was to determine whether they produced a result that seemed to represent real-world transactions. The factors used for comparison were standard deviation of return and time, which are common to all of the models. In Figures 1 and 2 I have compared the DLOM resulting from application of each of the models to data obtained from the FMV Restricted Stock Studies. Figure 1 shows the results of the study for transaction occurring after 1996, roughly representing a one-year restriction period. Figure 2 shows results for transactions occurring before 1997, roughly representing a two-year restriction period. Each figure shows standard deviation of return on the *x*-axis and DLOM on the *y*-axis. Each graph shows the same general result.

Two of the models do not appear to produce results that fall within the data. The comparison shows that the Longstaff model results in an upper bound and not in a realistic determination of the discount. Further, this model results in DLOMs greater than 100% for higher volatilities. The Tabak model does not compare well to the FMV data. For most of the range of the data, this model results in a discount greater than that shown in the study. In addition, the results from the study quickly approach discounts of 100%.

The other three models result in discounts that fall within the data. This tends to indicate that these models may be useful in representing the DLOM for one- and two-year periods.

Each of the models requires inputs in addition to time and standard deviation of the security's return.[9] These inputs were varied within a reasonable range. Although the specific discount computed at each range varies, the general conclusions presented here are not changed by alternate computations.

An additional test of the models is conducted by computing the results of the model over time using as an input a standard deviation of return that might be expected for valuations of smaller restricted or closely held companies. The results are shown in Figure 3, in which the standard deviation of return of the security that is used is 60%. This level of volatility is well within the range of the data from the FMV study, and such a level of volatility could well be expected for many small public companies.

Several issues arise with the Chaffe model when the results are examined over time. As the time to sale gets longer, the resulting DLOM stops increasing and actually gets smaller. For example, for a non-dividend-paying stock at a volatility of 60% and a time of seven years, the model produces a discount of 35%. Thereafter, the discount remains relatively constant for a period and then declines slightly. At 10 years the discount is reduced to about 34.4%. Such a result is counterintuitive. It is to be expected that the discount would continue to increase as the expected time to sale increases. An additional issue with this model is shown in Figure 4. The discount is higher when the interest rate (risk-free rate) is lower. This result does not seem reasonable. As the interest rate increases, it would be expected that the present value of future proceeds would decrease resulting in a higher discount. Thus the expectation is the opposite of the actual result from the model.

The performance of the Longstaff model over time shows that it indicates a DLOM of more than 100% at a relatively short time period. The fact that this model can result in a discount of greater than 100% raises the question of a theoretical problem. In addition, the short time period it takes for the results of this model to reach 100% limits its usefulness for many stocks.

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[9] Computational notes for Figures 1 and 2: In the Chaffee and Longstaff models, a risk-free rate of 5% and a 0% dividend rate are used. In the Meulbroek and Tabak models, a risk premium of 6%, a standard deviation of return of the market of 15%, and a beta of 1.0 are used. If these variables are changed, they do not change the conclusions presented in this article.

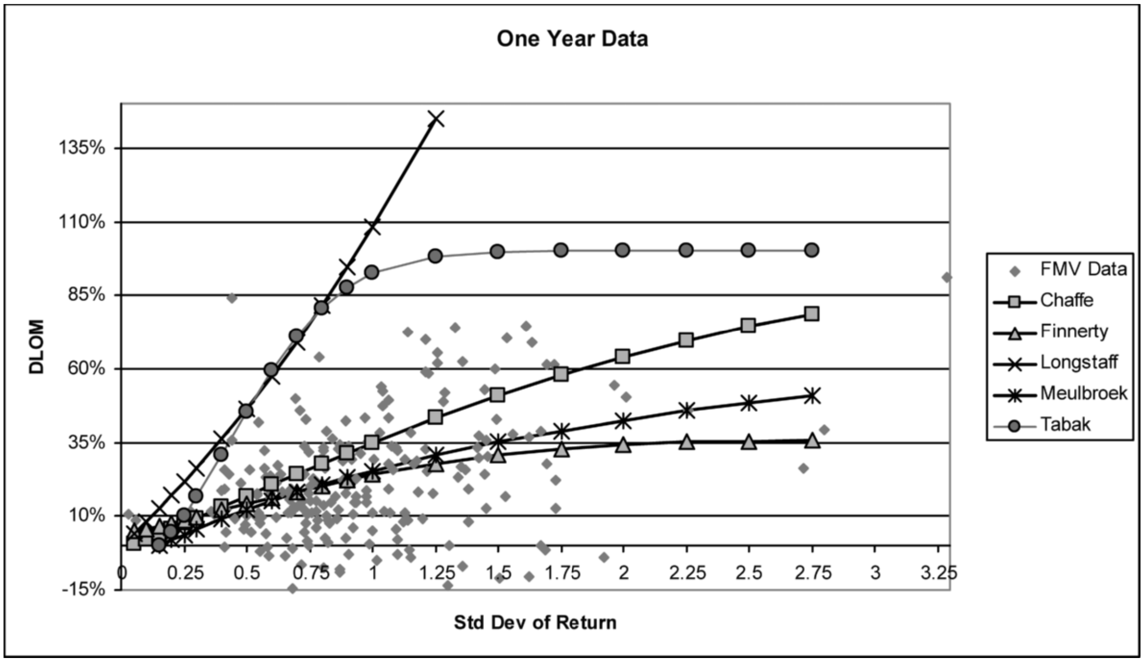

Figure 1

The Finnerty model has issues similar to the Longstaff model but less severe. It indicates a discount greater than 100% at about seven years. The fact that this model can result in a discount of greater than 100% raises the question of a theoretical problem. However, as discussed above, in shorter time periods, the model produces results that fall within the range of the FMV data.

The Meulbroek model behaves in a way over time that is in agreement with a common sense expectation; that is, as the period of time increases, the discount increases. However, it approaches but never exceeds

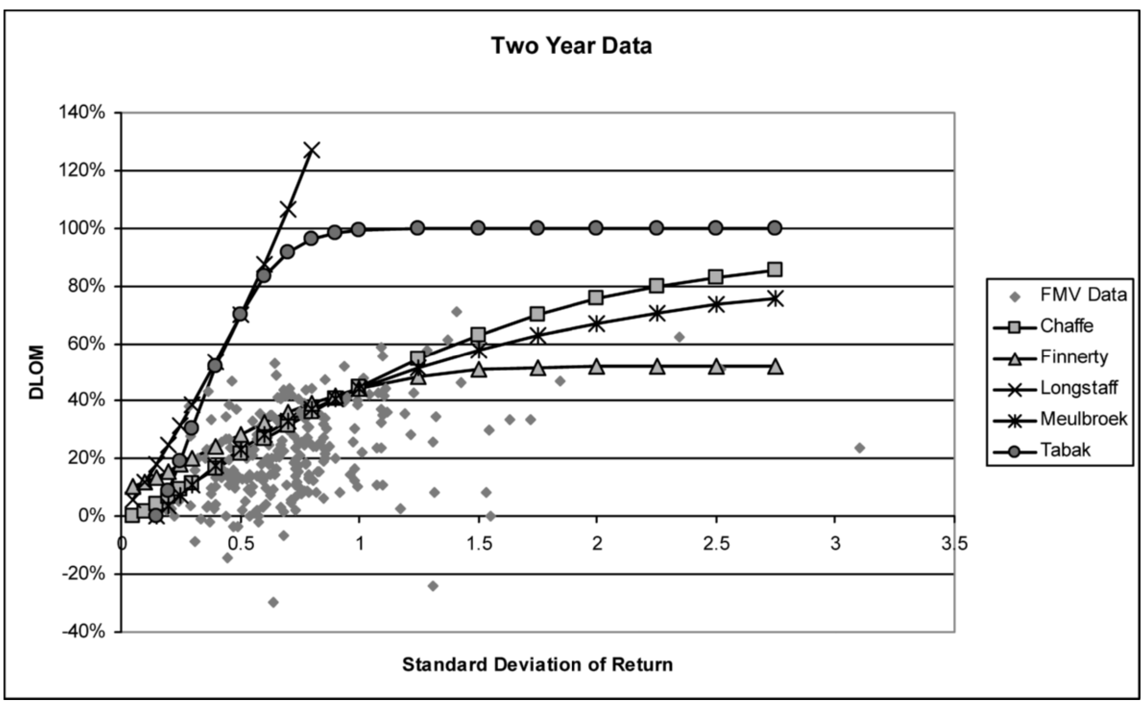

Figure 2

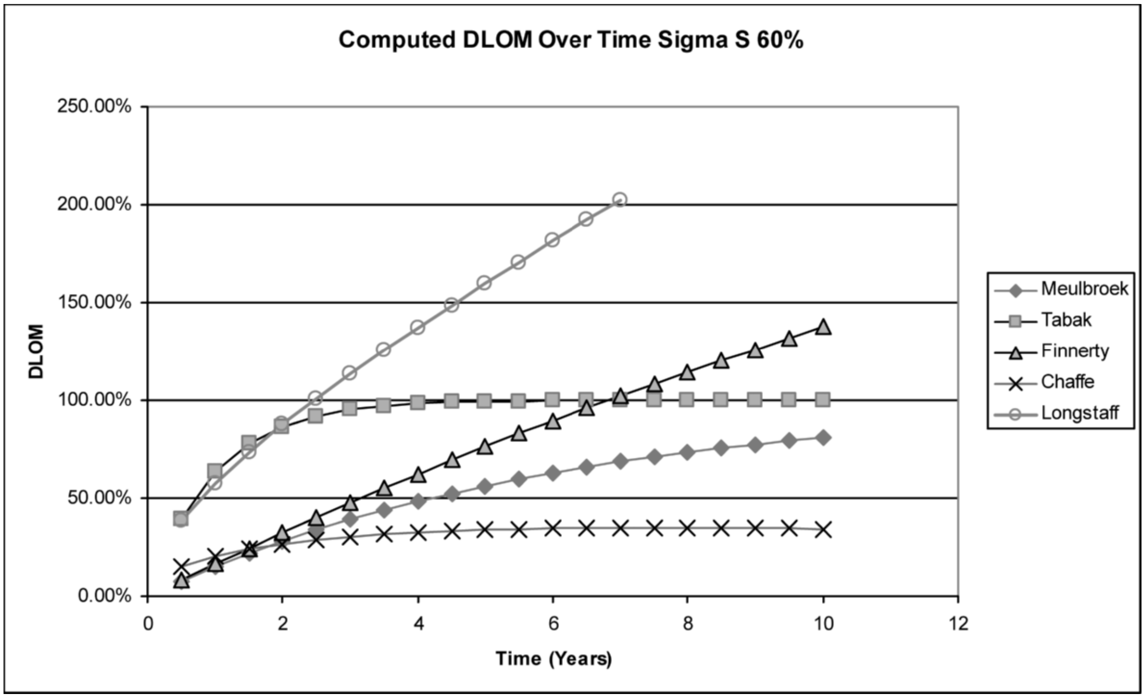

Figure 3

100% as long as the total beta is greater than CAPM beta.

The Tabak model results in a discount that increases rapidly at first and then quickly flattens out to indicate a discount of 100%.

Additional Notes about Theoretical Difficulties

All of these five models use volatility as an input. However, it is a well-known problem in using volatility to judging investments that desirable volatility in the

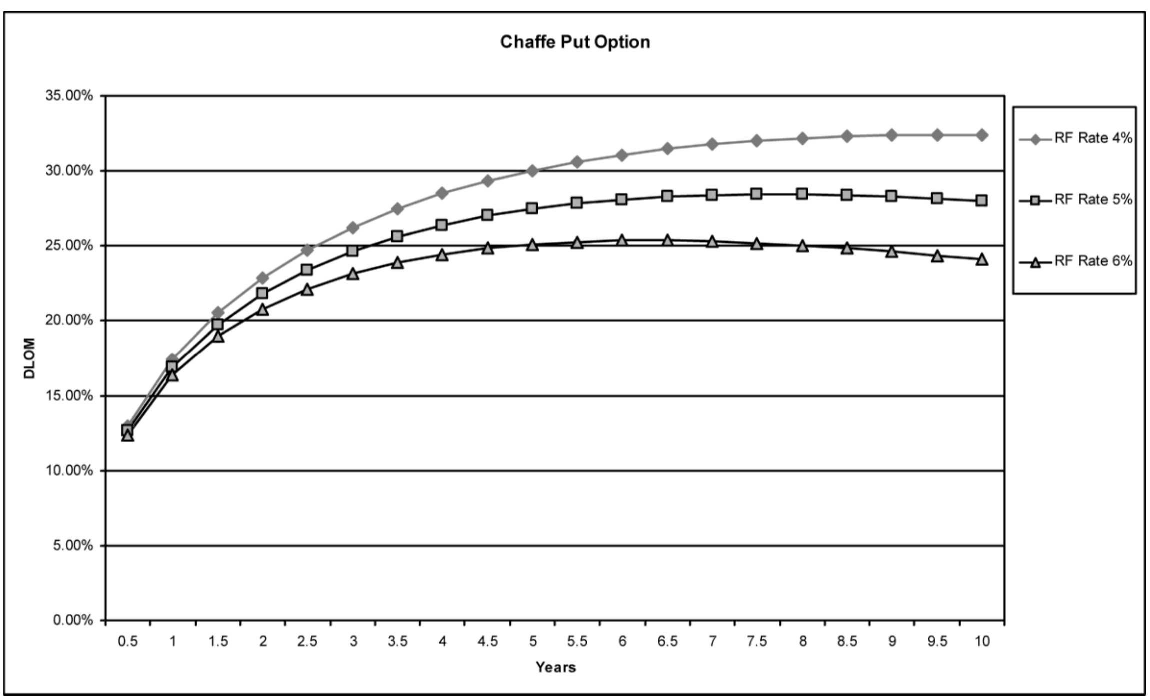

Figure 4

form of growth is not distinguished from total volatility. A stock with a history of a growth of return may appear volatile when judged against an average value. However, the apparent volatility is increased by the desirable growth. None of these models specifically account for this issue

In the CAPM-based models, those of Meulbroek, and Tabak, the impact of the risk-free cost of money is not considered. These models take the ability to invest in a portfolio of stock as a standard of comparison. This is a good standard for pensions in which it is likely that one would want to invest the funds in such a portfolio. However, for the sale of a single stock, this standard of comparison may not be as valid because the seller may merely want the money to spend or may want to invest in a greater or lesser risk investment. in such a case, the cost of money should be considered.

In the form presented above, each of the models studied relies on an explicit period of time to sale. However, in determining the DLOM, the time to sale is unknown and cannot be known until the sale is complete. Therefore, in using these models, it would be appropriate to combine results computed over a period of times with a probability of sale function over that period.

Even though there may be theoretical issues with a model, that does not mean that the model is useless. A model may produce results that are a reasonable representation of the real world within a certain range. Thus, it may provide an indication of the DLOM that may be considered by the valuator. For example, the Chaffe and Finnerty models produce results that appear representative of the data at short time periods even thought their results may be questioned over longer periods of time.

Conclusion

In this article five mathematical models that can be used to compute a DLOM are compared to restricted stock discounts to study how well each model performs in comparison to real-world data. It is found that the Chaffe, Finnerty, and Meulbroek models provide data that fall within the scatter of real-world data at one- and two-year time horizons. The Longstaff and Tabak models provide results that generally exceed the data.

The performance of the models is also studied over time. It is found that the Tabak and Longstaff models provide values that respectively equal or greatly exceed 100% at relatively short time frames. The Finnerty model also provides a result that exceeds 100% but at a longer time frame. Models providing results that equal or exceed 100% may have a theoretical problem because it seems logical that a discount would not reduce a value to zero or less. The Chaffe model is found to produce values that decline over time after reaching a peak. This also does not appear logical because it is to be expected that the DLOM will continue to increase as the expected time to sale increases. Further, the Chaffe model provides a result in which the discount is higher if the interest rate is lower, all other factors held constant. This does not appear to be logical. As the interest rate increases, it is expected that the DLOM would increase. The Meulbroek model produces a result over time that appears to be reasonable from a common sense standpoint.

Based on these results, the use of the Longstaff and Tabak models seems to have limited usefulness in most situations. The Chaffe and Finnerty models have some usefulness in shorter time frames but have limited usefulness as the expected time to sale increases. The Meulbroek model appears to be the most useful in both short- and long-term periods. However, the Meulbroek model uses the market portfolio as a standard of comparison, which limits the usefulness of the model in situations such as those involving portfolios of securities. In addition, it does not explicitly consider the important factors of growth and basic time value of money.

Finally, it is important to take note of the scatter of the FMV data around the factors of time and volatility. This scatter indicates that there are additional factors impacting the discount. Accordingly, the mathematical models studied here cannot be used as the sole indication of the DLOM. Qualitative factors must also be taken into account in using the models and reaching a judgment regarding the amount of the discount.

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